**Lecture 4.**

**Functions and limits. Theorem (the limit of function is unique). Properties of the limits.**

**Definition of a function**. If to every value of a variable , which belongs to some collection (set) , there corresponds one and only one finite value of the quantity then is said to be a *function* (single-valued) of or a *dependent variable* defined on the set , is the argument or *independent variable*. The fact that is a function of is expressed in brief form by the notation

 If to every value of , belonging to some set E there corresponds one or several values of the variable then is called a *multiple-valued function* of defined on From now on we shall use the word “function” only in the meaning of a single-valued function, if not otherwise stated.

**The domain of a function.** The collection of values of for which the given function is defined is called the *domain* of this function. In the simplest cases, the domain of a function is either a closed interval which is the set of real numbers that satisfy the inequalities or an open interval which is the set of real numbers that satisfy the inequalities Also possible is a more complex structure of the domain of function.

**Inverse function.** If the equation may be solved uniquely for the variable that is, if there is a function x such that

then the function x or, in standard notation, is the *inverse* of Obviously, that is the function is *the inverse* of (and vise versa).

 In the general case, the equation defines a multiple-valued inverse function such that for all that are values of the function

**Composite and implicit functions.** A function of defined by a series of equalities where u etc., is called a *composite function.* A function defined by an equation not solved for the dependent

variable is called an *implicit function*. For example, defines as an implicit function of

**Limit of a function**

 , where  is a domain of the function and  is limit point of .

**Definition.** We say that ****, if  there exists a such that if then (Figure 1).



Figure 1.

**Definition 2. (**according to Cauchy, i.e., “ definition of limit).

   ,   .

**Definition 3. (**according to Cauchy, i.e., “ definition of limit).

  ,   .

 **Definition . (** “neighborhood”definition of limit).

 **    **

 **Definition . (** “neighborhood”definition of limit).

 **    **

**Properties of the limits:**

Let be a real number, and suppose that 

That is, the limits exist and have values respectively. Then:

1. 

 ( the limit of a sum is the sum of the limits).

1. 

( the limit of a difference is the difference of the limits).

1. 

( the limit of a product is the product of the limits).

1. 

provided  ( the limit of a quotient is the quotient of the limits, provided the limit of the denominator is not zero).

1. 

provided ****, if  is even. (the limit of an nth root is the nth root of the limits).

1. 

(A constant factor can be moved through a limit symbol).